Discussion section week 7 - 11 oct 19

p. R-19 #2

STAT 7
7 oct 19

2(a)(i) $\sum_{i=1}^{3} 1 = 1 + 1 + 1 = 3$

2(a)(ii) $\sum_{i=1}^{n} 1 = \underbrace{1 + 1 + \dots + 1}_{n}$

$= n$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}^{y} \; n$$

① mean $\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$

2(a)(iii) $\sum_{i=1}^{5} i = 1 + 2 + \dots + 5$

$= 15$

2(a)(iv)

$$\left(\sum_{i=1}^{n} y_i \right) - \left(\sum_{j=1}^{n} y_j \right)$$

$$= (y_1 + y_2 + \dots + y_n) - (y_1 + y_2 + \dots + y_n)$$

$$= 0$$

2(a)(v) $\sum_{i=1}^{n} (y_i + c) = (y_1 + c) + (y_2 + c) + \dots + (y_n + c)$

$$= (y_1 + y_2 + \dots + y_n) + (\underbrace{c + c + \dots + c}_{n})$$

$$\boxed{= \left(\sum_{i=1}^{n} y_i\right) + nc = \sum_{i=1}^{n} (y_i + c)}$$

so $$\frac{1}{n}\sum_{i=1}^{n} y_i + \frac{\cancel{n}c}{\cancel{n}} = \frac{1}{n}\sum_{i=1}^{n} (y_i + c)$$

$$\boxed{\underbrace{\frac{1}{n}\sum_{i=1}^{n} (y_i + c)}_{\bar{y}_{new}} = \underbrace{\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right)}_{\bar{y}_{old}} + c}$$

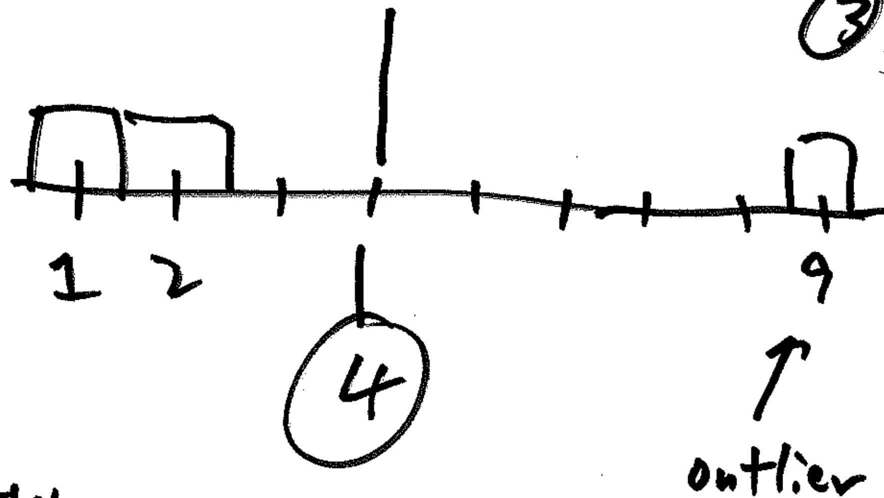
$$\begin{array}{ccc} \text{old} & & \text{new} \\ \begin{bmatrix} y_1 \\ y_2 \\ \\ y_n \end{bmatrix} & \xrightarrow{\text{add } c} & \begin{bmatrix} y_1 + c \\ y_2 + c \\ \vdots \\ y_n + c \end{bmatrix} \end{array}$$

mean $\bar{y} = \bar{y}_{old}$ mean $\frac{1}{n}\sum_{i=1}^{n} (y_i + c) = \bar{y}_{new}$

$= \bar{y}_{old} + c$

hwk 1 3(a)(i)

$$\begin{array}{c} \text{old} \\ \begin{bmatrix}1\\2\\9\end{bmatrix} \end{array} \quad n=3$$



mean $\bar{y} = 4 = \bar{y}_{old}$

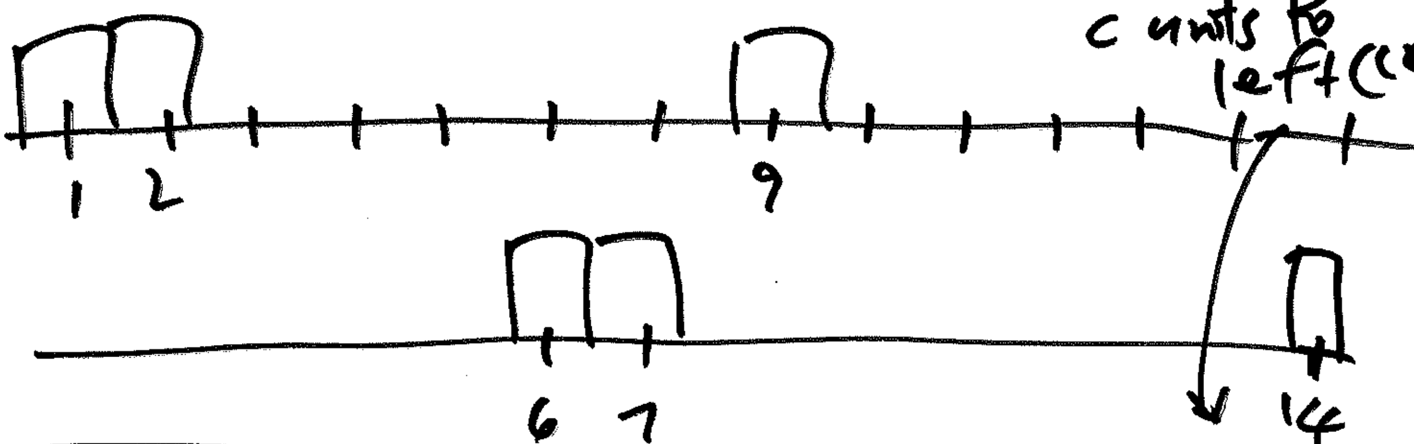
quant.

discrete

$$\begin{bmatrix}1\\2\\9\end{bmatrix} \xrightarrow[5=c]{\text{add}} \begin{array}{c} \text{new} \\ \begin{bmatrix}6\\7\\14\end{bmatrix} \end{array}$$

mean $\bar{y}_{new} = 9 = \bar{y}_{old} + c$

shifting c units to left ($c<0$)



or right ($c>0$)

2(b) show that

$$\sum_{i=1}^{n} (c\, y_i) = c\left(\sum_{i=1}^{n} y_i\right)$$

$$\sum_{i=1}^{n} (c y_i) = c y_1 + c y_2 + \dots + c y_n$$

$$= c\underline{(y_1 + \dots + y_n)} = c\left(\sum_{i=1}^{n} y_i\right)$$

✓

$$\frac{1}{n}\sum_{i=1}^{n} (c y_i) = c\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right)$$

$$\begin{array}{ccc} \text{old} & & \text{new} \\ \begin{bmatrix} y_1 \\ y_2 \\ y_n \end{bmatrix} & \xrightarrow[\text{by } c]{\text{multiply}} & \begin{bmatrix} c y_1 \\ c y_2 \\ \vdots \\ c y_n \end{bmatrix} \end{array}$$

mean $\bar{y}_{old}$

$$= \frac{1}{n}\sum_{i=1}^{n} y_i$$

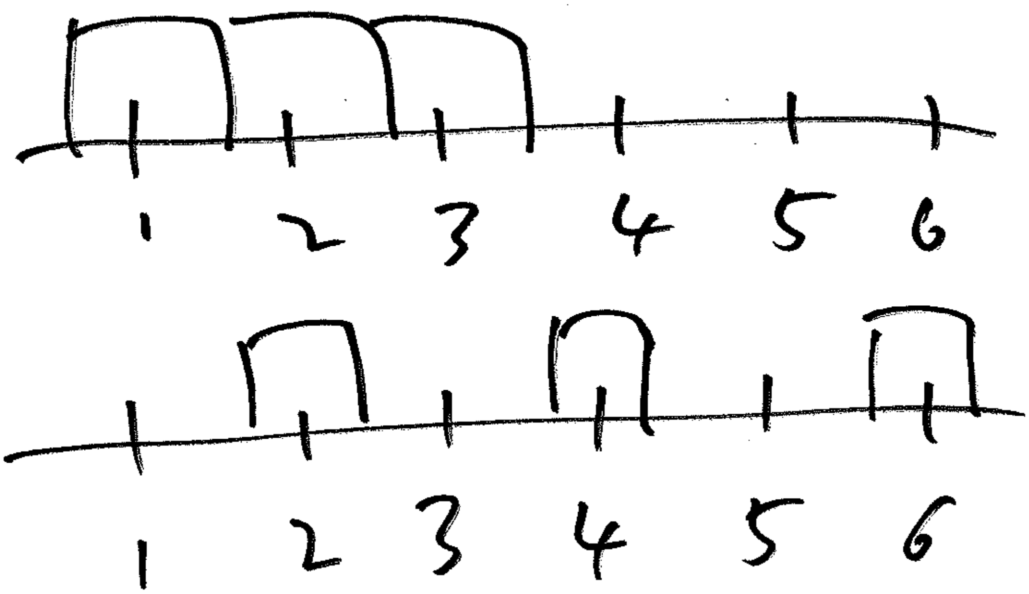
mean $\bar{y}_{new} = \frac{1}{n}\sum_{i=1}^{n} (c y_i)$

$$= c\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right)$$

$$\boxed{\bar{y}_{new} = c \cdot \bar{y}_{old}}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \; n = 3 \; \xrightarrow[\text{by } 2]{\text{mult.}} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

mean $\bar{y}_{old} = 2$ mean $\bar{y}_{new} = 4 = 2 \cdot \bar{y}_{old}$



R-29 #1

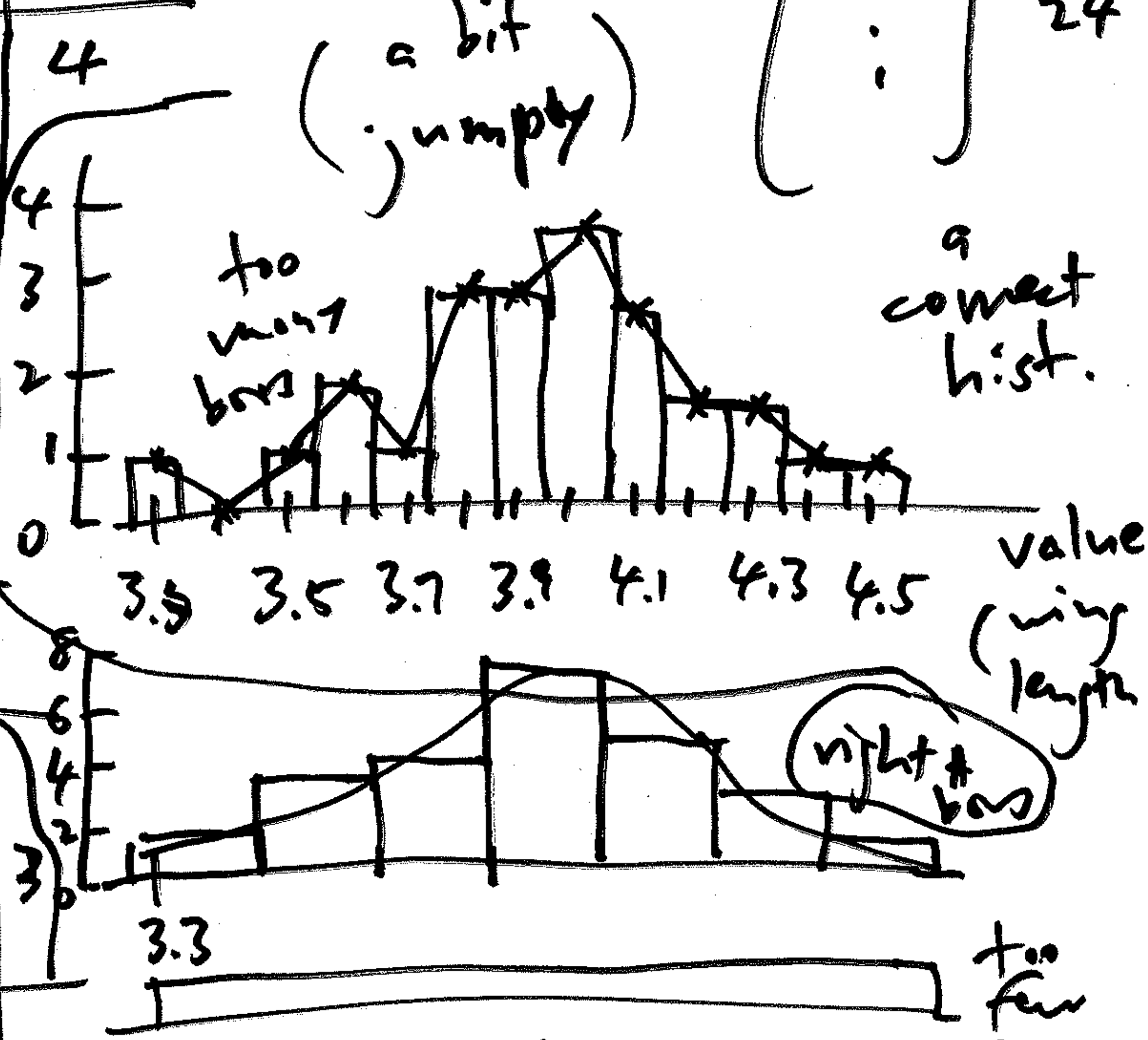
⑥

value	raw freq. (count)	
3.3	1	1
3.4	0	
3.5	1	3
3.6	2	
3.7	1	4
3.8	3	
3.9	3	7
4.0	4	
4.1	3	5
4.2	2	
4.3	2	3
4.4	1	
4.5	1	1

n=24

(wing length)

$$y = \begin{bmatrix} 3.9 \\ 4.4 \\ 3.5 \\ \vdots \end{bmatrix} \quad n = 24$$



$4.5 - 3.3 = 1.2$

$(1.2)(10) + 1 = 13$

p. R-19 #2

2(a)(i) $\sum_{i=1}^{3} 1 = 1+1+1$

$= 3$

2(a)(ii) $\sum_{i=1}^{n} 1 = \underbrace{1+1+\dots+1}_{n}$

$= n$

$y \leftarrow$ quant

y_1
y_2
$\vdots$
y_n

n

mean $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$

2(a)(iii) $\sum_{i=1}^{5} i = 1+2+3+4+5$

$= 15$

2(a)(iv) $\left(\sum_{i=1}^{n} y_i\right) - \left(\sum_{j=1}^{n} y_j\right)$

$= (y_1 + y_2 + \dots + y_n) - (y_1 + y_2 + \dots + y_n)$

$= 0$

2(a)(v) $\sum_{i=1}^{n} (y_i + c) = *$

$* = (y_1 + c) + (y_2 + c) + \dots + (y_n + c)$

$$= (y_1 + y_2 + \cdots + y_n) + (c + c + \cdots + c)$$

$$n$$

$$= \boxed{\left(\sum_{i=1}^{n} y_i\right) + nc \overset{(*)}{=} \sum_{i=1}^{n} (y_i + c)}$$

$$\frac{1}{n}\left[\left(\sum_{i=1}^{n} y_i\right) + nc\right] = \frac{1}{n}\sum_{i=1}^{n}(y_i + c)$$

$$\frac{1}{n}\sum_{i=1}^{n} y_i + \frac{1}{n}(nc) = \frac{1}{n}\sum_{i=1}^{n}(y_i + c)$$

$$\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right) + c$$

$$2(3+5) = 2\cdot 3 + 2\cdot 5$$

$$\begin{array}{ccc} \text{old} & & \text{new} \\ \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix} n = 3 & \xrightarrow[1]{\text{add}} & \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix} \end{array}$$

mean $\bar{y}_{old} = 3$ mean $\bar{y}_{new} = 4$

quant ✓

old $\qquad\qquad\qquad\qquad$ new

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \xrightarrow[c]{\text{add}} \begin{pmatrix} y_1+c \\ y_2+c \\ \vdots \\ y_n+c \end{pmatrix}$$

mean $\bar{y}_{old} = \frac{1}{n}\sum_{i=1}^{n} y_i$ $\qquad$ mean $\bar{y}_{new} = \frac{1}{n}\sum_{i=1}^{n} (y_i+c)$

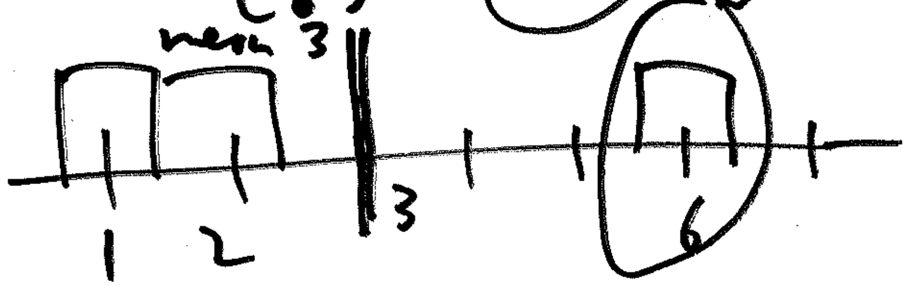
$= \bar{y}_{old} + c$

shifting c units to left $(c<0)$ or right $(c>0)$

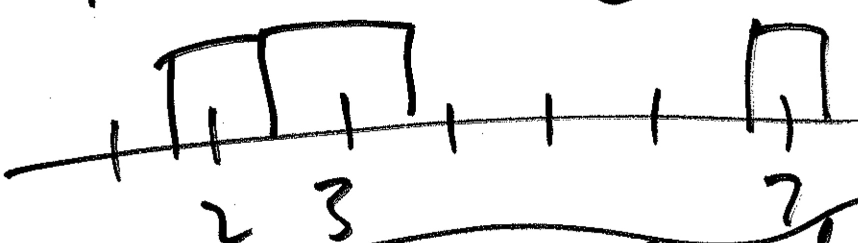
$\begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}$ old

outlier

mean 3



new



add constant → spread stays same

2(b) $\sum_{i=1}^{n} (c y_i) = c y_1 + c y_2 + \cdots + c y_n$

$= c(y_1 + \cdots + y_n) = c\left(\sum_{i=1}^{n} y_i\right)$

$$\frac{1}{n}\sum_{i=1}^{n}(cy_i) = c\left(\frac{1}{n}\sum_{i=1}^{n}y_i\right)$$

$$\begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix} \xrightarrow[\text{by } 2]{\text{mult.}} \begin{bmatrix} 2 \\ 4 \\ 12 \end{bmatrix}$$

mean 3 mean 6

old new

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \xrightarrow[\text{by } c]{\text{mult.}} \begin{bmatrix} cy_1 \\ cy_2 \\ \vdots \\ cy_n \end{bmatrix}$$

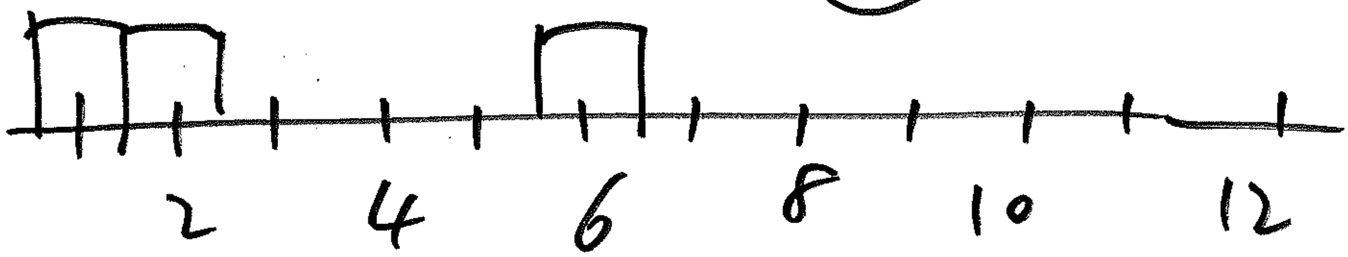
mean $\bar{y}_{old}$

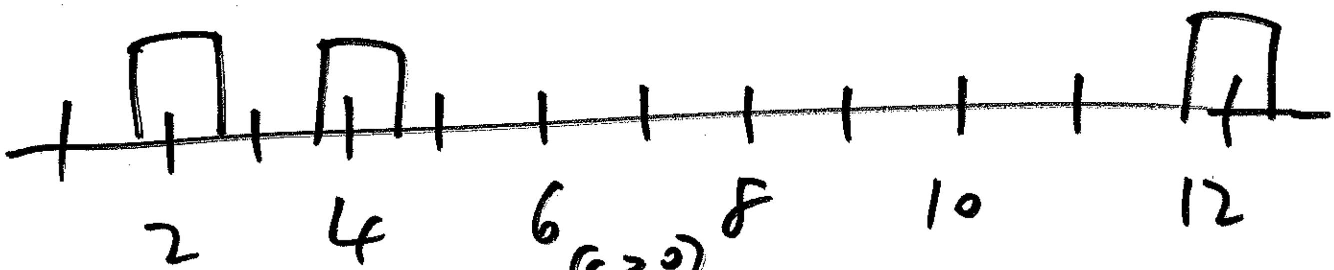
$$= \frac{1}{n}\sum_{i=1}^{n}y_i$$

mean $\bar{y}_{new} = \frac{1}{n}\sum_{i=1}^{n}(cy_i)$

$$= c\left(\frac{1}{n}\sum_{i=1}^{n}y_i\right)$$

$$\bar{y}_{new} = c \cdot \bar{y}_{old}$$





(c > 0)

mult. by (c > 1) → spread ↑

c < 1 ↓