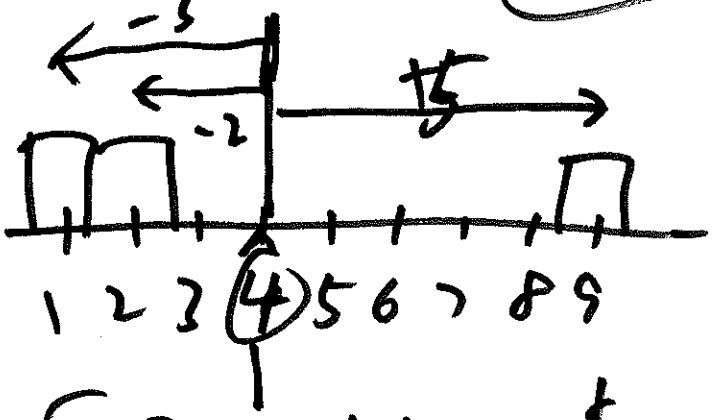
Discussion Section
week of 14-18 Oct 19

R-29 # 3

Stat 7
14 Oct 19
Q



y

$$\begin{pmatrix} y_1 \\ y_2 \\ y_n \end{pmatrix} \quad n=3 \xrightarrow{\text{subtract } 4} \begin{pmatrix} -3 \\ -2 \\ +5 \end{pmatrix} \xrightarrow{|\cdot|} \begin{pmatrix} |-3| \\ |-2| \\ |+5| \end{pmatrix}$$

mean $\bar{y} = 4$

mean 0

mean $\frac{10}{3} = 3.3$

$$\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \xrightarrow[\text{by } \bar{y}]{\text{subtract}} \begin{pmatrix} y_1 - \bar{y} \\ \vdots \\ y_n - \bar{y} \end{pmatrix} \xrightarrow{|\cdot|} \begin{pmatrix} |y_1 - \bar{y}| \\ \vdots \\ |y_n - \bar{y}| \end{pmatrix}$$

mean $\bar{y}$

mean 0

$$\text{MAD} = \frac{1}{n} \sum_{i=1}^{n} |y_i - \bar{y}|$$

Laplace (~1790) → ↑ mean absolute deviation

Gauss (~1800) instead of $|\cdot|$, use $(\cdot)^2$

$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}$ $\xrightarrow[4]{\text{subtract}}$ $\begin{bmatrix} -3 \\ -2 \\ +5 \end{bmatrix}$ $\xrightarrow{\text{square}}$ $\begin{bmatrix} (-3)^2 = 9 \\ (-2)^2 = 4 \\ (+5)^2 = 25 \end{bmatrix}$ ②

mean 4

mean $\frac{38}{3} \approx 12.7$

$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$ $\uparrow n$ $\xrightarrow[\bar{y}]{\text{subtract}}$ $\begin{bmatrix} y_1 - \bar{y} \\ \vdots \\ y_n - \bar{y} \end{bmatrix}$ $\xrightarrow{\text{square}}$ $\begin{bmatrix} (y_1 - \bar{y})^2 \\ (y_2 - \bar{y})^2 \\ (y_n - \bar{y})^2 \end{bmatrix}$

mean $\bar{y}$

mean $\frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2$

problem: units wrong for

easy fix: take $\sqrt{\ }$ at end

$\sqrt{\frac{38}{3}} \approx \sqrt{12.7}$

≈ 3.6

$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2}$

sample standard deviation (s)

$$(\text{sample } s)^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

= sample variance

"Gaussian dist."

"normal" curve

(units wrong, makes some math easy)

K. Pearson (~1900):

"SD"

$f(y)$

SD's

100%

$\bar{y}$ $y \rightarrow$

approx. histogram on density scale

$$f(y) = \frac{1}{s\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{y-\bar{y}}{s}\right)^2\right]$$

de Moivre (1710)

$= g(c)$

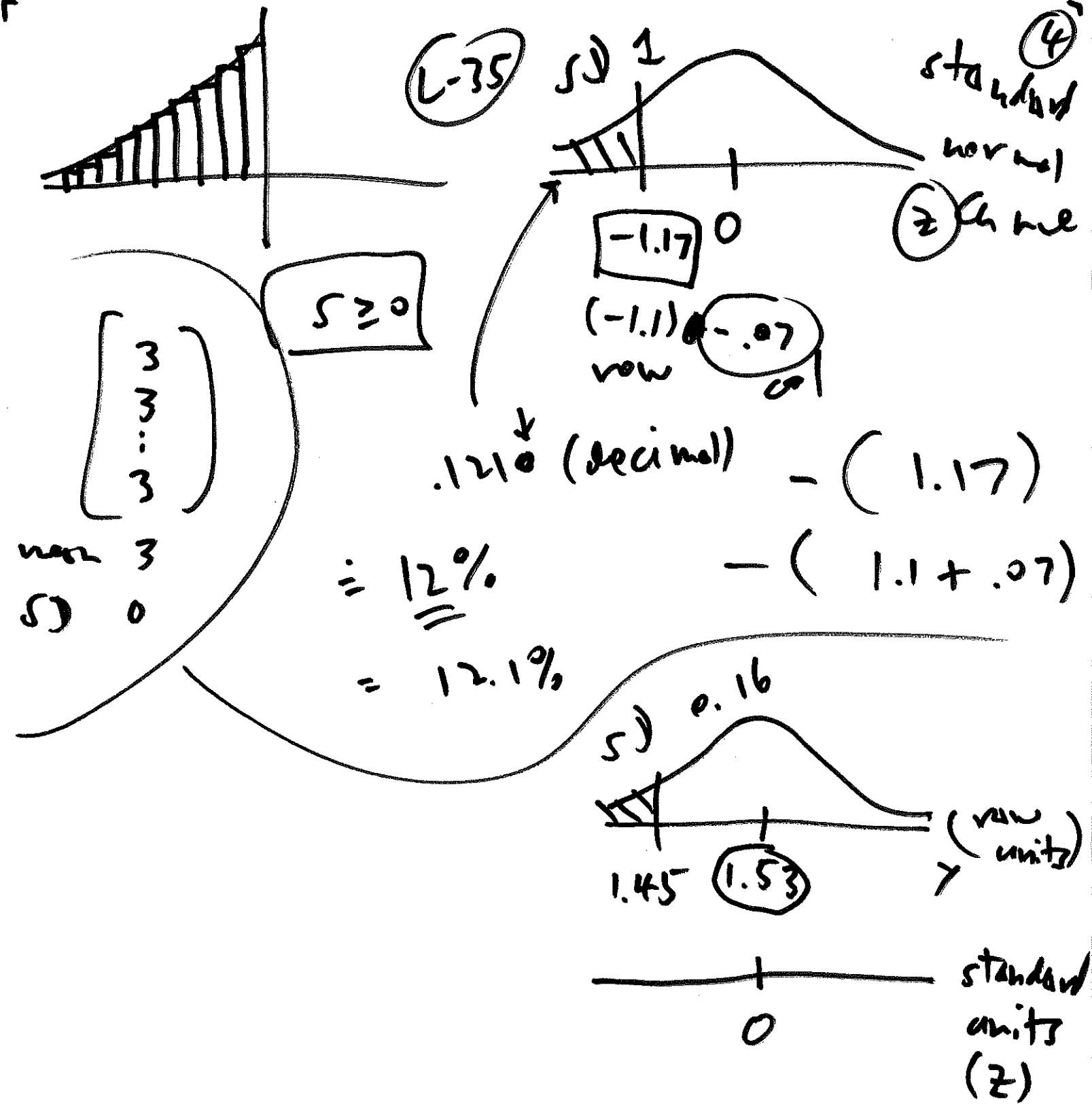
$\int_{-\infty}^{c} e^{-x^2}\,dx$

what % of frogs had e.a. ≤ 1.45?

SD 0.16

1.45 1.53

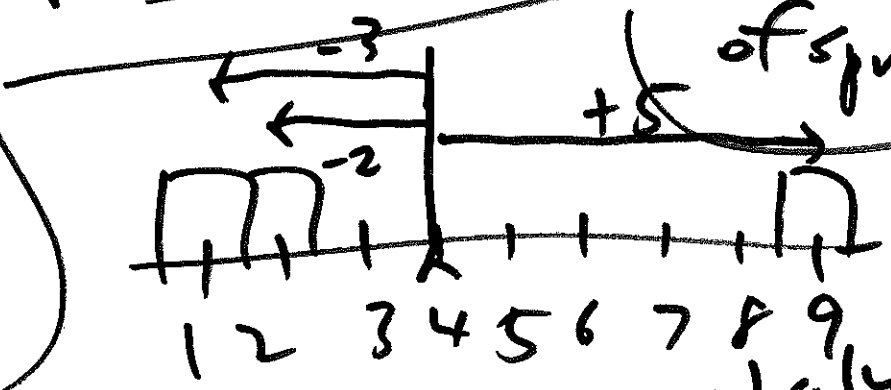
hist. of enzyme activity (e.a.)


L-35
SD 1
standard normal curve
z
-1.17
0
(-1.1) + .07
row
.1210 (decimal)
= 12%
= 12.1%
- (1.17)
- (1.1 + .07)
S ≥ 0
[3 3 ⋮ 3]
row 3
SD 0
SD 0.16
1.45
1.53
(raw units)
X
standard units (Z)
0

Discussion Section, week of 14-18 Oct 19

p. R-(29) #3 (5)

measures of spread



take absolute value

$$\begin{bmatrix} \$1 \\ \$2 \\ \$9 \end{bmatrix} \uparrow n=3 \downarrow \xrightarrow[4]{\text{subtract}} \begin{bmatrix} 1-4=-3 \\ 2-4=-2 \\ 9-4=+5 \end{bmatrix} \rightarrow \begin{bmatrix} |-3|=\$3 \\ |-2|=\$2 \\ |+5|=\$5 \end{bmatrix}$$

mean $\$4$ — mean 0 — mean $\frac{\$10}{3} = \$3.3$ = MAD ⊛

deviations from mean

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \uparrow n \downarrow \xrightarrow[\bar{y}]{\text{subtract}} \begin{bmatrix} y_1-\bar{y} \\ y_2-\bar{y} \\ \vdots \\ y_n-\bar{y} \end{bmatrix} \rightarrow \begin{bmatrix} |y_1-\bar{y}| \\ \vdots \\ |y_n-\bar{y}| \end{bmatrix}$$

mean $\bar{y}$ — mean 0 — mean ⊛

$$MAD \quad \frac{1}{n}\sum_{i=1}^{n} |y_i - \bar{y}| = \text{mean absolute deviation}$$

(Laplace: ~1785)

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \to \begin{bmatrix} -3 \\ -2 \\ +5 \end{bmatrix} \xrightarrow{\text{square}} \begin{bmatrix} (-3)^2 = 9 \\ (-2)^2 = 4 \\ (+5)^2 = 25 \end{bmatrix}$$

Gauss (~1785) mean $\frac{38}{3} = 12.7$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \to \begin{bmatrix} y_1 - \bar{y} \\ \vdots \\ y_n - \bar{y} \end{bmatrix} \xrightarrow{\text{square}} \begin{bmatrix} (y_1 - \bar{y})^2 \\ \vdots \\ (y_n - \bar{y})^2 \end{bmatrix}$$

mean $\bar{y}$

$\sqrt{12.7}$ ← mean $\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2$ (**)

$= 3.6$

take $\sqrt{\ }$ at end to get correct units

for a now-obscure reason (mystery explained in week 5 or so), people divide in (**) not by n but by $(n-1)$:

$$\to \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2 = s^2$$

sample variance

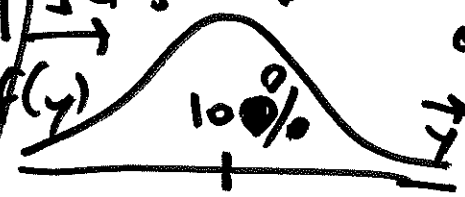
sample standard deviation (SD)

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

↑ lower case

normal curve

de Moivre (~1710) $f(y)$



on density scale, relative frequency = area under curve

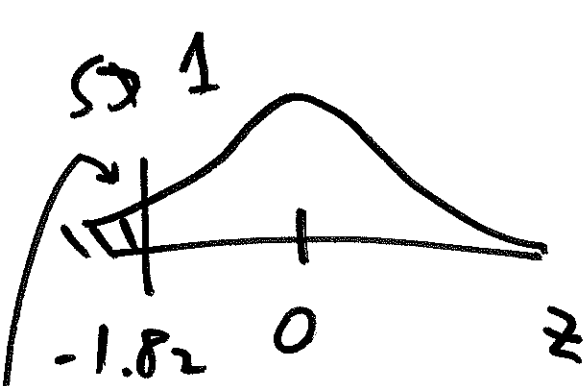
"Gaussian distribution"

on the density scale

$$f(y) = \frac{1}{s\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{y-\bar{y}}{s}\right)^2\right]$$

Q: what (%) of frogs in sample had enzyme ≤ 1.37 ?

$$\int_{-\infty}^{c} e^{-x^2}\,dx = ?$$



standard normal curve

(8)

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

≥ 0

(35)

$-(1.82)$

$-(1.8 + .02)$

rows columns

.0344

↑

always

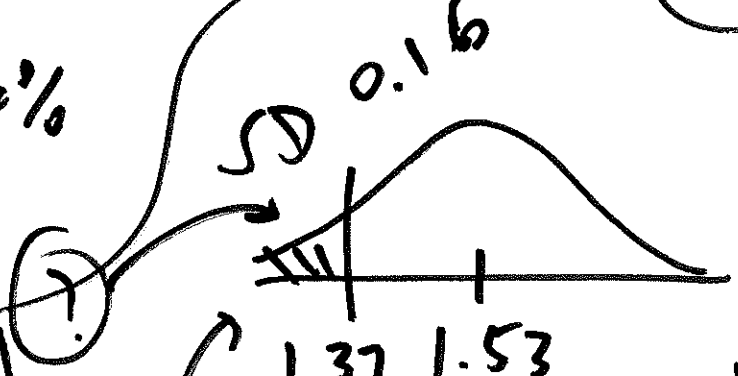
decimals

$\doteq 3.44\% \doteq 3.4\%$

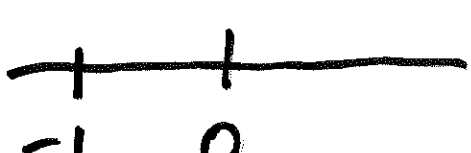
$$\begin{bmatrix} 6 \\ 6 \\ \vdots \\ 6 \end{bmatrix}$$

mean 6

SD 0



how many SDs is 1.37 away from 1.53?



$$z = \frac{y - \bar{y}}{s}$$

$.1587 \doteq 16\%$