Stats 7 - class 9 10-24

L-97 Equally likely model (ELM): if you can enumerate {all the ways the repeatable phenomenon you're thinking about can come out} in such a way that all of these possible outcomes are equally likely, then for any event A

$$P(A) = \frac{\text{number of outcomes favorable to } A}{\text{total number of possible outcomes}}$$

ex (pop) $\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow{\text{at random}}$ (sample) $\begin{bmatrix} \overline{Y} \end{bmatrix}$ ELM? = yes

$P(Y = 9) = \frac{1}{3}$

$P(Y \text{ is odd}) = \frac{2}{3}$

ex p (any single child is normal) = $\frac{1}{4}$ = 25%

ELM? = yes

P(" is a carrier) = $\frac{2}{4}$ = 50%

P(" T-S) = $\frac{1}{4}$ = 25%

P(1 or more have T-S in family of n) = p_n

as n↑, p_n↑

* qualitative reasoning

A B * T/F statements

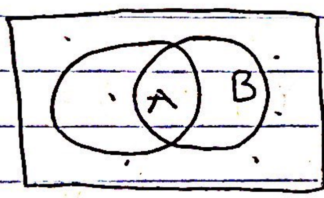
(A and B)
(A or B)
(not A)
(B given A)

$P(A \text{ or } B) \overset{?}{=} P(A) \; ? \; P(B)$

$P(\text{not } A) \; ? \; P(A)$

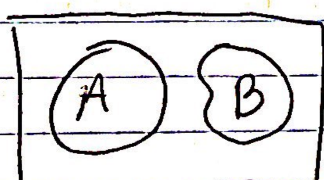
$P(A \text{ and } B) \; ? \; P(A) \; ? \; P(B)$

OR



$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

general addition rule for or



$P(A \text{ or } B) = P(A) + P(B)$

special case addition rule for or with no overlap

if A and B have no overlap, A and B are mutually exclusive

→ $P(A \text{ and } B) = 0\%$

For any event (T/F statement)

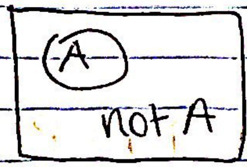
A, $\quad {}^{(0)}\, 0\% \leq P(A) \leq 100\% \, {}^{(1)}$

(A is true, a certain event)

(A is false)
(A is an impossible event)

Scanned with CamScanner

Negative Propability: meaningless



$$P(A) + P(\text{not } A) = P[A \text{ or } (\text{not } A)] = 1 = 100\%$$

useful ★ $P(A) = \underset{\uparrow 100\%}{1} - P(\text{not } A)$

ex $\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow[\text{random}]{\text{at}} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$

$$P(Y_1 = 9 \text{ and } Y_2 = 9) = ?$$

Case 1 at random with replacement = independent identically distributed (IID) sampling

$P(Y_1 = 9) \underset{\text{IID}}{=} \frac{1}{3}$

$P(Y_2 = 9) \underset{\text{IID}}{=} \frac{1}{3}$

	Draw 1		
1	(1,1)	(1,2)	(1,9)
2	(2,1)	(2,2)	(2,9)
9	(9,1)	(9,2)	(9,9)
	1	2	9

ELM? = yes

$$P(Y_1 = 9 \text{ and } Y_2 = 9) = \frac{1}{9}$$

Theory: $P(Y_1 = 9) \cdot P(Y_2 = 9) = \frac{1}{3} \cdot \frac{1}{3}$
(works for IID)

Case 2: at random without replacement = simple random sampling (SRS)

		Draw 2		
		1	2	9
draw 1	1	~~(1,1)~~	(1,2)	(1,9)
	2	(2,1)	~~(2,2)~~	(2,9)
	9	(9,1)	(9,2)	~~(9,9)~~

$P_{SRS}(Y_1 = 9) = \frac{2}{6}$?

$P_{IID}(Y_1 = 9) = \frac{1}{3}$

ELM? yes

$$P_{SRS}(Y_2 = 9) = \frac{2}{6} = \frac{1}{3}$$

$$P_{IID}(Y_2 = 9) = \frac{1}{3}$$

$$P_{SRS}(Y_1 = 9 \text{ and } Y_2 = 9) = 0$$

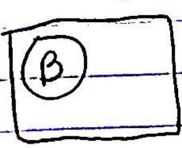
$$\neq P_{SRS}(Y_1 = 9) \cdot P_{SRS}(Y_2 = 9)$$

$$\neq \frac{1}{3} \cdot \frac{1}{3} \neq \frac{1}{9}$$

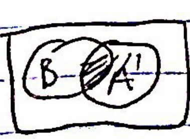
Theory doesn't work

Conditional Probability

P(B given A) = ?



$P(B) = \dfrac{B}{\boxed{1}}$



$P(B \text{ given } A) = \dfrac{\text{(B and A)}}{A}$

$$P(B \text{ given } A) = \begin{cases} \dfrac{P(A \text{ and } B)}{P(A)} & \text{if } P(A) > 0 \\ \text{undefined} & \text{if } P(A) = 0 \end{cases}$$

$P(B|A)$

| (given)

general rule for and

$$\star\ P(A \text{ and } B) = P(A) \cdot P(B|A)$$
$$= P(B) \cdot P(A|B)$$

ex $P_{SRS}(Y_1 = 9 \text{ and } Y_2 = 9) =$

$$P_{SRS}(Y_1 = 9) \cdot P(Y_2 = 9 \mid Y_1 = 9)$$
$$= \frac{1}{3} \cdot 0 = 0$$

R-37

def A, B are independent — Baysian ✗ info about A doesn't change chance of B and vice versa

↳ $P(A \text{ and } B) = P(A) \cdot P(B)$ frequency

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

Special Case Rule for and under independence:

A, B independent

$$\rightarrow P(A \text{ and } B) = P(A) \cdot P(B|A)$$
$$= P(A) \cdot P(B)$$

independent → IID

ex $P($ 1 or more T-S babies in family of 5, both parents are carriers$)$

$= 1 - P(\text{no T-S})$

$= 1 - P(\text{not T-S on } 1^{st} \text{ and } \ldots \text{ not T-S on } 5^{th})$

$\overset{IID}{=} 1 - P(\text{not T-S on } 1^{st}) \ldots P(\text{not T-S on } 5^{th})$

$= 1 - (1 - \frac{1}{4}) \ldots (1 - \frac{1}{4})$

$= 1 - (1 - \frac{1}{4})^5 = .76 = \underline{76\%}$