PROBABILITY, EQUALLY LIKELY MODEL (10/24/19)

Read: DD (A)
Chpt 1-3 (B)
Chpt 1-9 LN pg 1-120 1

Quiz 3 due tomorrow night
Quiz 4 due next tues 29 oct
HW 2 due next weds 30 oct

L-97 → (Pascal, Fermat)

Equally likely model (ELM): If you can enumerate {all the ways the repeatable phenomenon you're thinking about can come out} in such a way that all of these possible outcomes are equally likely then for any event A

$$P(A) = \frac{\text{\# of outcomes favorable to A}}{\text{total \# of possible outcomes}}$$

ex (pop) $\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow{\text{at random}}$ (sample) $\begin{bmatrix} Y \end{bmatrix}$

?ELM = yes

$P(Y = 9) = \frac{1}{3}$

$P(Y \text{ is odd}) = 2/3$

ex P(any single child is normal) = $1/4 = 25\%$

ELM? yes

P(" is a carrier) = $2/4 = 50\%$ P(" T-S) = $1/4 = 25\%$

P(1 or more have T-S in family of n) = P_n

as $n\uparrow$, $P\uparrow$

*qualitative reasoning

A B * T/F statements

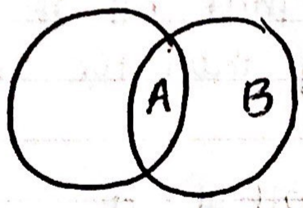
(A and B)
(A or B)
(not A)
(B given A)

P(A or B) $\overset{?}{=}$ P(A) ? P(B)

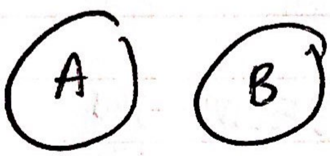
P(not A) ? P(A)

P(A and B) ? P(A) ? P(B)

OR



P(A or B) = P(A) + P(B) − P(A and B)
general addition rule for or



P(A or B) = P(A) + P(B)

special case addition rule for or w/ no overlap
if A and B have no overlap, A and B are mutually exclusive → P(A and B) = 0%

For any event (T/F statements)

A, $\quad 0\% \le P(A) \le 100\%$

0%: (A is false) (A is an impossible event)

100%: (A is true, a certain event)

Negative Probability: meaningless

A

not A

$P(A) + P(\text{not } A) = P[A \text{ or } (\text{not } A)] = 1 = 100\%$

useful ☆ $P(A) = 1 - P(\text{not } A)$

1: 100%

ex

$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow[\text{random}]{\text{at}} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$

$P(X_1 = 9 \text{ and } X_2 = 9) = ?$

$\boxed{\text{case 1}}$ at random w/ replacement = independent identically distributed (IID) sampling

$P(X_1 = 9) = \frac{1}{3}$ IID

$P(X_2 = 9) = \frac{1}{3}$ IID

Draw 1

1	(1,1)	(1,2)	(1,9)
2	(2,1)	(2,2)	(2,9)
9	(9,1)	(9,2)	(9,9)
	1	2	9

$$P(X_1 = 9 \text{ and } X_2 = 9) = 1/9$$

theory: $P(\Sigma_1 = 9) \cdot P(X_2 = 9) = 1/3 \cdot 1/3$
(works for IID)

$\boxed{\text{CASE 2}}$ at random w/o replacement = simple random sampling (SRS)

draw 1 \ Draw 2	1	2	9
1	(1,1)	(1,2)	(1,9)
2	(2,1)	(3,2)	(2,9)
9	(9,1)	(9,2)	(9,9)

$P_{SRS}(X_1 = 9) = \frac{2}{6}$

$P_{IID}(X = 9) = \frac{1}{3}$

ELM? yes

$$P_{SRS}(X_2 = 9) = \frac{2}{6} = \frac{1}{3}$$

$$P_{IID}(X_2 = 9) = \frac{1}{3}$$

$$P_{SRS}(X_1 = 9 \text{ and } X_2 = 9) = 0$$

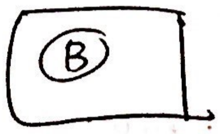
$$\neq P_{SRS}(X_1 = 9) \cdot P_{SRS}(X_2 = 9)$$

$$\neq \frac{1}{3} \cdot \frac{1}{3} \neq \frac{1}{9}$$

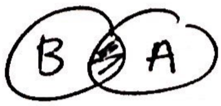
Theory doesn't work

Conditional Probability

P(B given A) = ?



$P(B) = \frac{B}{\square}$



$P(B \text{ given } A) = \frac{\equiv}{A}$ ← (B and A)

$P(B \text{ given } A) = \frac{P(A \text{ and } B)}{P(A)}$ if $P(A) > 0$

$P(B \mid A)$ ↑ (given) — undefined if $P(A) = 0$

General Rule for And

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$
$$P(B) \cdot P(A \mid B)$$

ex $P_{SRS}(Y_1 = 9 \text{ and } Y_2 = 9) = P_{SRS}(Y_1 = 9) \cdot P(Y_2 = 9 \mid Y_1 = 9)$

$= \frac{1}{3} \cdot 0 = 0$

R-37

A, B are independent ↙ * info about A doesn't change chance of B and vice versa

↳ $P(A \text{ and } B) = P(A) \cdot P(B)$

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

special case rule for and under independence:

A, B independent

$\rightarrow P(A \text{ and } B) = P(B) \cdot P(B|A)$
$= P(A) \cdot P(B)$

IID

independent

ex P(1 or more T-S babies in family of 5, both parents are carriers)

$= 1 - P(\text{no T-S})$

$= 1 - P(\text{not T-S on 1st and not T-S on 5th})$

$\overset{IID}{=} 1 - P(\text{not T-S on 1st}) \ldots P(\text{not T-S on 5th})$

$= 1 - (1 - \frac{1}{4}) \ldots (1 - \frac{1}{4})$

$= 1 - (1 - \frac{1}{4})^5 = .76 = 76\%$