STAT 7 7 Nov 19

this time: statistical inference for means &
next time: proportions

read: LN pp. L-127 → L-160

take home midterm due **Sun** night

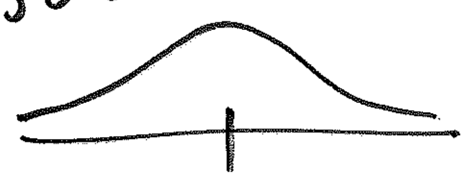
quiz 6 due next Tue night

square root law

$$SE_{\underline{\underline{IID}}}(\bar{y}) = \frac{\sigma}{\sqrt{n}}$$

$SE_{IID}(\bar{y})$ = our uncertainty about the pop. mean μ

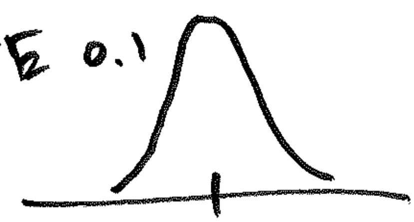
SD 0.2 = SE



3.8
EV

hist. of $\bar{y}$, $n = \underline{1}$

SE 0.1

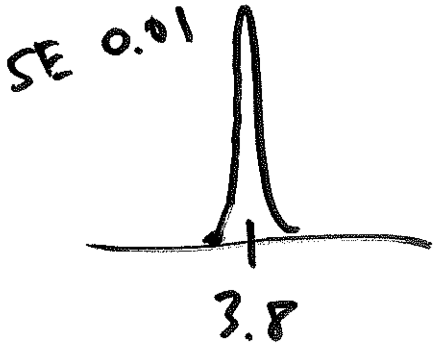


3.8
EV

hist. of $\bar{y}$, $n = \underline{\underline{4}}$

As $n\uparrow$, $SE(\bar{y})\downarrow$ but only at a $\sqrt{n}$ rate:

to cut the SE in half you need to quadruple the sample size



hist. of $\bar{y}_1$
$n = 400$

L - 139

L - 143
Jmp

pop $\leftrightarrow$ broadest scope of valid <u>generalizability</u> outward from the dataset

probability

pop.
general
whole

deduction $\rightarrow$

sample
specific
part

$\leftarrow$ induction

statistical
inference

each number in <u>pop.</u> dataset is around $\underline{\mu \doteq \bar{y}}$

give or take about $\underline{\sigma \doteq s}$

pop.
all crabs of same species similar to sampled crabs in all relevant ways

stat. inf.

sample
the observed crabs

imag. data

body temp.

N = ? (big)

relevant ways: actual, like SRS ≐ IID

body temp.

25.8
|
25.4

n = 25

all possible values of $\bar{y}$

25.0
24.8
⋮

M → ∞

mean $\bar{y} = 25.0\,^\circ C$

SD $s = 1.34\,^\circ C$

sample hist

pop. mean $\mu = ?$

pop. SD $\sigma = ?$

pop hist

hyp. IID

n = 25

mean $\bar{y} = ?$
(ex. 24.8)

long run mean	expected value of $\bar{y}$ $= \mu$
long run SD	standard error of $\bar{y} \doteq 0.27$

long run hist.

$$\left(\text{expected value of } \bar{y}\right) = \left(\text{EV of } \bar{y}\right) = E_{IID}(\bar{y}) = \mu$$

95% conf. int. for μ

$\bar{y}$

inferential summary

pop.	unknown pop. quantity of main interest	μ = pop. mean body temp. at equilibrium
sample	estimate of μ	$\bar{y} = 25.0^\circ C$ $(n = 25)$
inference	give or take for $\bar{y}$ as est. of μ	$\widehat{SE}(\bar{y}) \doteq 0.27^\circ C$
	95% CI for μ	$\bar{y} \pm t^{0.95}_{n-1} \frac{s}{\sqrt{n}} = (24.5, 25.6)^\circ C$

on basis of data, we think that μ is around $\bar{y} = 25.0^\circ C$, give or take about $\widehat{SE}(\bar{y}) = 0.27^\circ C$

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each measurement in ~~data~~ sample set is around $\bar{y} = 25.0^\circ C$, give or take about $s = 1.34^\circ C$

each number ($\bar{y}$) in the imaginary dataset is around μ ________,

give or take about $\frac{\sigma}{\sqrt{n}}$ ________

$$\begin{pmatrix} \text{estimated standard error of } \bar{y} \end{pmatrix} = \begin{pmatrix} \hat{SE} \text{ of } \bar{y} \end{pmatrix} = \hat{SE}_{IID}(\bar{y}) = \frac{\cancel{\sigma}\, s}{\sqrt{n}}$$

← "hat"

now $\hat{SE}(\bar{y}) = \frac{s}{\sqrt{n}} = \frac{1.34°C}{\sqrt{25}} \doteq 0.27°C$

Fisher → $\boxed{95\%}$ confidence interval for μ (CI)

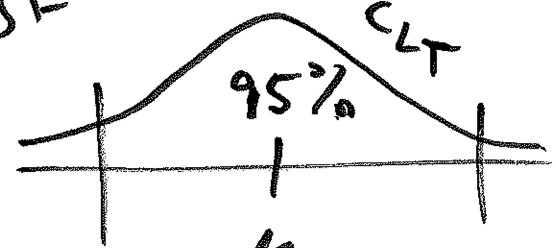
confidence level

{ Neyman (1935)

(1908) 6

Gosset
"Student"

$\hat{SE}$ 0.27°C



long-run hist of $\bar{y}$ if σ known

SU (Z)

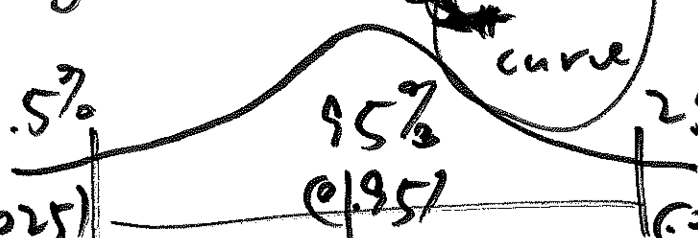
−1.96 1.96

degrees of freedom

$t_{24} = t_{n-1}$ curve

long-run hist of $\bar{y}$, accounting for uncertainty about σ

$\hat{SE}$ 0.27°C



$\mu - 2.026\,\hat{SE}$ μ EV $\mu + 2.026\,\hat{SE}$

−2.064 2.064

L-142

$$P_F\left(\mu - 2.064\,\hat{SE} < \bar{y} < \mu + 2.064\,\hat{SE}\right) = 95\%$$

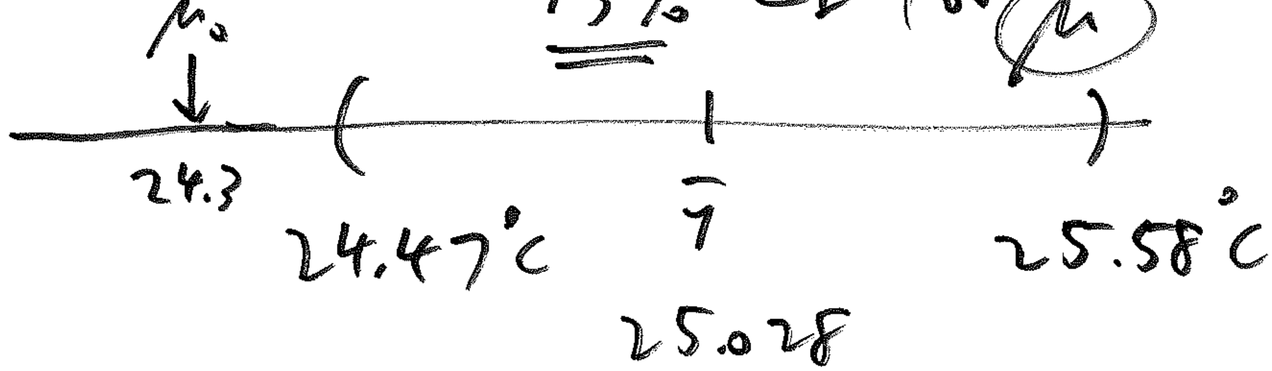
$$P_F\left(\underbrace{\bar{y}}_{\text{random}} - 2.064\,\widehat{SE} < \underbrace{\mu}_{\text{fixed}} < \underbrace{\bar{y}}_{\text{random}} + 2.064\,\widehat{SE}\right) = 95\%$$

95% CI for μ

$$\bar{y} \pm t^{0.95}_{n-1} \frac{s}{\sqrt{n}}$$

sample mean $\pm$ (t #) (estimated SE of $\bar{y}$)

95% CI for μ



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theory: $\mu = \mu_0 = 24.3°C$ because μ_0 is not

$\therefore$ 95% CI, data do not support theory