Quiz 3 due tomorrow night.

Due next Thursday homework 2 due next Wed 29 Oct
(take home midterm handed out next Thursday in class, due about 5 days later)

likely model (ELM):

\[
P(Y = 9) = \frac{1}{3}
\]

\[
P(\hat{Y} = 0.66) = \frac{2}{3}
\]
\[ P(\text{any single child is male}) = \frac{1}{4} \]

\[ P(HH) = 25\% \]

\[ P(\text{at least 2 boys}) = \frac{1}{4} \]

\[ P(\text{1 or more T-J in family of 4}) = p_n \]

\[ \text{as } n \uparrow \text{, } p_n \uparrow \]

Qualitative reasoning

\[ (A \text{ and } B) \quad (\neg A) \quad (B \text{ given } A) \]

\[ (A \text{ or } B) \]

T/F statements
\[ P(A \text{ or } B) = P(A) + P(B) \]

\[ P(\text{not } A) = 1 - P(A) \]

\[ P(A \text{ and } B) = P(A) \cdot P(B) \]

**Venn diagram**

\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

**General addition rule for or**

\[ P(A \text{ or } B) = P(A) + P(B) \]

**Special addition rule for or with no overlap**

\[ P(A) + P(B) \]

if \( A, B \) have no overlap

\( A, B \) are mutually exclusive

\[ + P(A \text{ and } B) = 0\% \]

\[ R \left( \text{(no overlap)} \right) \]
for any event (T/F statement)

\[ 0 \% \leq P(A) \leq 100\% \]  \hspace{1cm} (1)

(A is false) \hspace{1cm} (A is true)

(A is impossible) \hspace{1cm} (A is a certain event)

\[ \neg A + P(\neg A) = 1 = 100\% \]

\[ P(A \text{ or } \neg A) \]

\[ P(A) = 1 - P(\neg A) \]

(100\%)
\[
\begin{bmatrix}
1 \\
2 \\
9
\end{bmatrix}
\]

\[
\begin{bmatrix}
?_1 \\
?_2
\end{bmatrix}
\]

\[P(\Psi_1 = 9 \text{ and } \Psi_2 = 9) = \frac{1}{9}\]

Case 1

at random with replacement

- independent identically distributed (IID) sample

\[
\begin{array}{ccc}
1 & 2 & 9 \\
1 & (1,1) & (1,2) & (1,9) \\
2 & (2,1) & (2,2) & (2,9) \\
9 & (9,1) & (9,2) & (9,9) \\
\end{array}
\]

\[P(\Psi_2 = 9) = \frac{1}{3}\]

\[P(\Psi_2 = 9) = 1 - \frac{3}{9} = \frac{2}{3}\]

\[P(\Psi_1 = 9) = \frac{1}{3}\]

\[P(\Psi_1 = 9 \text{ and } \Psi_2 = 9) = \frac{1}{9}\]
case 2) at random without replacement

= simple random sampling (SRS)

\[
P_{SRS}(\Xi_1 = 9) = \frac{2}{6}
\]
\[
P_{SRS}(\Xi_2 = 9) = \frac{2}{6}
\]
\[
P_{\Xi_2}(\Xi_1 = 9) = \frac{1}{3}
\]
\[
P_{\Xi_1}(\Xi_2 = 9) = \frac{1}{3}
\]

\[
P(\Xi_1 = 9 \text{ and } \Xi_2 = 9) = 0
\]

\[
\frac{1}{3} \cdot \frac{1}{3} \neq \frac{1}{9}
\]

theory doesn't work for SRS
conditional probability

\[ P(B \text{ given } A) = ? \]

\[ P(B) = \frac{\text{\# of } B}{\text{\# of } \Omega} \]

\[ P(B \text{ given } A) = \frac{P(A \text{ and } B)}{P(A)} \]

\[ P(B \text{ given } A) \text{ is } \frac{P(A \text{ and } B)}{P(A)} \]

\[ \text{def. } P(B \text{ given } A) = \frac{P(A \text{ and } B)}{P(A)} \]

\[ \text{if } P(A) > 0 \]

\[ \text{undefined if } P(A) = 0 \]
The general rule for \( A \text{ and } B \) is:

\[
P(A \text{ and } B) = P(A) \cdot P(B|A) = P(B) \cdot P(A|B)
\]

Given:

\[
P(X_1 = 9 \text{ and } X_2 = 9) = \frac{1}{3}
\]

Since \( X_1 = 9 \) and \( X_2 = 9 \) are independent, we have:

\[
\frac{1}{3} = \frac{1}{6} \cdot \frac{1}{6}
\]

Defining:

A, B are independent

Frequency:

\[
P(A \text{ and } B) = P(A) \cdot P(B)
\]
Special case rule for **and** under independence

\[ A, B \text{ independent} \]

\[ p(A \text{ and } B) = p(A) \cdot p(B|A) \]

\[ = p(A) \cdot p(B) \]

\( \frac{1}{5} \) or more T-S babies in family of 5, both parents carriers

\[ = 1 - p(\text{no T-S}) \]

\[ = 1 - p(\text{not T-S}) \cdot p(\text{not T-S}) \cdot p(\text{not T-S}) \cdot p(\text{not T-S}) \cdot p(\text{not T-S}) \]

IID

\[ = 1 - \left( \text{not T-S} \right) \cdot \left( \text{not T-S} \right) \cdot \left( \text{not T-S} \right) \cdot \left( \text{not T-S} \right) \cdot \left( \text{not T-S} \right) \]
\[ 1 - \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{4}\right) \cdots \left(1 - \frac{1}{4}\right) \]
\[ = 1 - \left(1 - \frac{1}{4}\right)^5 = 0.76 \approx 76\% \]

[Handwritten note: take-home test]