this time: probability; next time: probability models for sums

read: DD (A) ch. 1-3; (B) ch. 1-9; LN pp. 1-126 ①

STAT 7 24 oct 19

quiz 3 due tomorrow night

quiz 4 due next Tue 29 oct

homework 2 due next wed 30 oct (may slip 2 days)

take home midterm handed out next Tue in class, due about 9 days later

L-97 → (Pascal, Fermat) (1650) equally likely model (ELM):

(pop) (sample) ELM? yes

$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}$ $\xrightarrow[\text{random}]{\text{at}}$ $\begin{bmatrix} Y \end{bmatrix}$

$$P(Y = 9) = \frac{1}{3}$$

$$P(Y \text{ odd}) = \frac{2}{3}$$

p(any single child is normal) $= \frac{1}{4}$

ELM?
yes

p(H H) $= 25\%$

p(______ carrier) $= \frac{2}{4}$

$= p$(H h) $= 50\%$

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p(______ T-S) $= \frac{1}{4}$ %

$= 25\%$

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p(1 or more T-S in family of n) $= p_n$

as $n \uparrow$ $p_n \uparrow$

qualitative reasoning

A B

T/F statements

(A and B)

(A or B)

(not A)

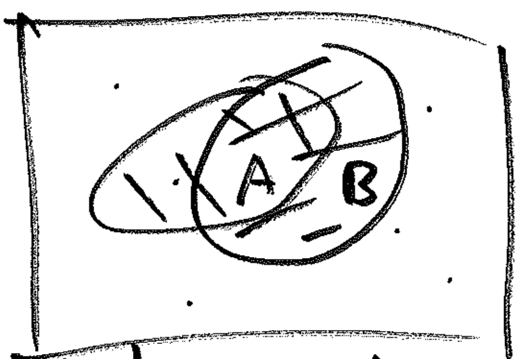
(B given A)

$P(A \text{ or } B) \; ? \; P(A) \; ? \; P(B)$

$P(\text{not } A) \; ? \; P(A)$

$P(A \text{ and } B) \; ? \; P(A) \; ? \; P(B)$

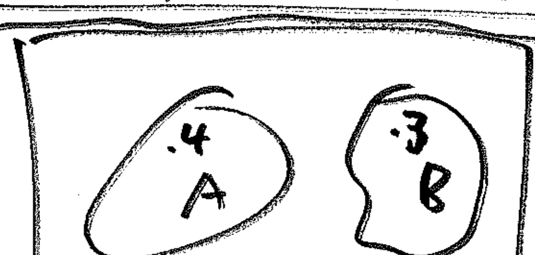
OR



Venn diagram

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

general addition rule for OR



$$P(A \text{ or } B) = P(A) + P(B)$$

Special case addition rule for OR with no overlap

if A, B have no overlap → R-37

A, B are mutually exclusive

→ $P(A \text{ and } B) = 0\%$

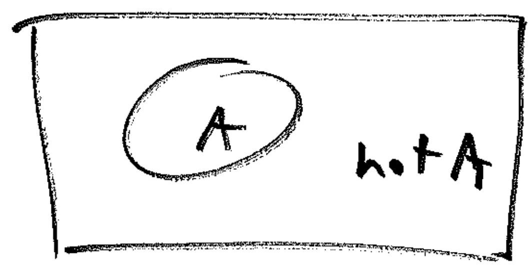
for any event (T/F statement)

A, $(0)\ 0\% \leq P(A) \leq 100\%\ (1)$

(A is false) or (A is impossible event)

(A is true) (A is a certain event)

negative probabilities are meaningless



$$\boxed{P(A) + P(\text{not } A) = 1} = 100\%$$

$$P[A \text{ or } (\text{not } A)]$$

$$\boxed{P(A) = 1 - P(\text{not } A)}$$

← really useful

(100%)

pop $\qquad\qquad$ sample

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow{\text{at random}} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$$

$$P(Y_1 = 9 \text{ and } Y_2 = 9) = ?$$

$\boxed{\text{Case 1}}$ at random with replacement

= independent identically distributed (IID) sampling

IID draw 2

draw 1	1	2	9
1	(1,1)	(1,2)	(1,9)
2	(2,1)	(2,2)	(2,9)
9	(9,1)	(9,2)	(9,9)

3×3 table

ELM?

yes ✓

yes ELM?

$$P(Y_1 = 9) = \frac{1}{3}$$

IID $= \frac{3}{9}$

$$P(Y_2 = 9) = \frac{1}{3}$$

IID $= \frac{3}{9}$

$$P(Y_1 = 9 \text{ and } Y_2 = 9) = \frac{1}{9}$$

theory works for IID

$$= P(Y_1 = 9) \cdot P(Y_2 = 9) = \frac{1}{3} \cdot \frac{1}{3}$$

case 2 at random without replacement

= simple random sampling (SRS)

SRS

$$\begin{array}{c|ccc}
 & & \text{draw 2} & \\
\text{draw 1} & 1 & 2 & 9 \\
\hline
1 & \cancel{(1,1)} & (1,2) & (1,9) \\
2 & (2,1) & \cancel{(2,2)} & (2,9) \\
9 & (9,1) & (9,2) & \cancel{(9,9)}
\end{array}$$

ELM? yes

$$P_{SRS}(Y_1 = 9) = \frac{2}{6}$$

$$P_{IID}(Y_1 = 9) = \frac{1}{3}$$

$$P_{SRS}(Y_2 = 9) = \frac{2}{6}$$

$$P_{IID}(Y_2 = 9) = \frac{1}{3}$$

$$P_{SRS}(Y_1 = 9 \text{ and } Y_2 = 9) = 0$$

$$\neq P_{SRS}(Y_1 = 9) \cdot P_{SRS}(Y_2 = 9)$$

$$\neq \frac{1}{3} \cdot \frac{1}{3} \neq \frac{1}{9}$$

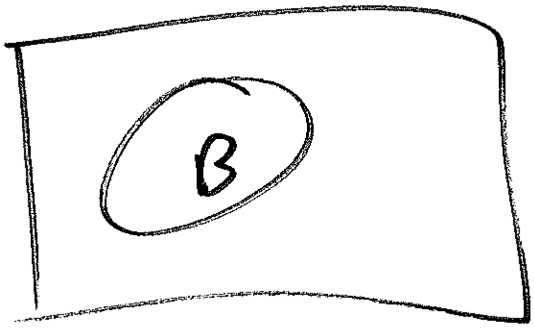
theory doesn't work for SRS

conditional probability

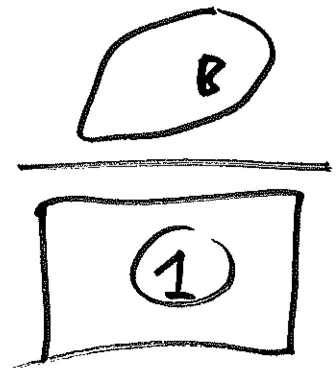
de Moivre (1705)

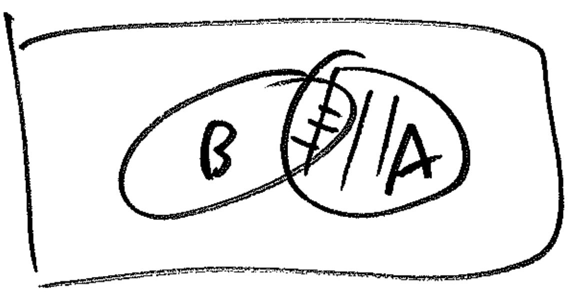
Bayes (1760)

P(B given A) = ?



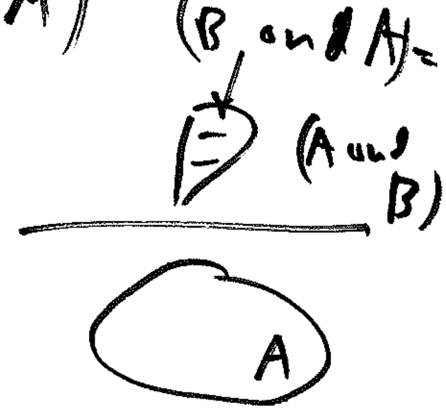
P(B) =





P(B given A)

=



def.

$$P(B \text{ given } A) = \begin{cases} \dfrac{P(A \text{ and } B)}{P(A)} & \text{if } P(A) > 0 \\ \text{undefined} & \text{if } P(A) = 0 \end{cases}$$

$= P(B \mid A)$

"given"

general rule for and

$$P(A \text{ and } B) = P(A)\cdot P(B|A)$$
$$= P(B)\cdot P(A|B)$$

$$P_{SRS}(\mathcal{I}_1 = 9 \text{ and } \mathcal{I}_2 = 9) =$$

$$P_{SRS}(\mathcal{I}_1 = 9)\cdot P(\mathcal{I}_2 = 9 \mid \mathcal{I}_1 = 9)$$

$$= \frac{1}{3} \cdot 0 = 0 \quad \checkmark$$

Bayesian } info about A does not change chances of B & vice verse

Def: A, B are independent

$\leftrightarrow$ $P(A \text{ and } B) = P(A)\cdot P(B)$

frequency

$$P(A \text{ and } B) = P(A)\cdot P(B|A)$$
$$= P(A)\cdot P(B)$$

⑨

special case rule for and under independence

A, B independent

$\rightarrow$ $P(A \text{ and } B) =$

$P(A) \cdot P(B|A)$

$= P(A) \cdot P(B)$

IID

independent

$P($1 or more T-S babies in family of 5, both parents carriers$)$

$= 1 - P(\text{no T-S})$

$= 1 - P\left(\begin{matrix}\text{not}\\ \text{T-S}\\ \text{on 1st}\end{matrix} \text{ and } \begin{matrix}\text{not}\\ \text{T-S}\\ \text{on 2nd}\end{matrix} \text{ and } \ldots \text{ and } \begin{matrix}\text{not}\\ \text{T-S}\\ \text{on 5th}\end{matrix}\right)$

IID

$= 1 - P\left(\begin{matrix}\text{not}\\ \text{T-S}\\ \text{on 1st}\end{matrix}\right) \cdot P\left(\begin{matrix}\text{not}\\ \text{T-S}\\ \text{on 2nd}\end{matrix}\right) \cdots P\left(\begin{matrix}\text{not}\\ \text{T-S}\\ \text{on 5th}\end{matrix}\right)$

$$= 1 - \left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{4}\right)\cdots\left(1 - \frac{1}{4}\right)$$

$$= 1 - \left(1 - \frac{1}{4}\right)^5 \doteq 0.76 \doteq \underline{\underline{76\%}}$$

take-home test